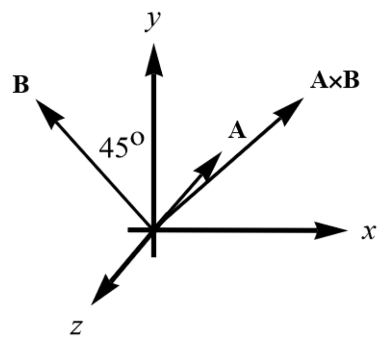
Exercise 1.52

The vector $\vec{A}$ is 3.50 cm long and is directed into this page. Vector $\vec{B}$ points from the lower right corner of this page to the upper left corner of this page. Define an appropriate right-handed coordinate system, and find the three components of the vector product $\vec{A} \times \vec{B}$, measured in cm^2. In a diagram, show your coordinate system and the vectors $\vec{A}$, $\vec{B}$, and $\vec{A} \times \vec{B}$?

Solution

Let the page be the xy-plane.



Since $\mathbf{B}$ goes from the bottom right to the top left, it forms an angle of 45° with the y-axis. Write formulas for the given vectors.

$$\mathbf{A} = 3.50\langle 0, 0, -1\rangle \text{ cm}$$

$$\mathbf{B} = |\mathbf{B}|\langle -\sin 45^\circ, \cos 45^\circ, 0\rangle \text{ cm}$$

The magnitude of $\mathbf{B}$ isn't given, so just leave it as $|\mathbf{B}|$. Calculate the cross product of $\mathbf{A}$ and $\mathbf{B}$.

$$\begin{aligned}
\mathbf{A} \times \mathbf{B} &= \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ 0 & 0 & -3.50 \\ -|\mathbf{B}|\sin 45^\circ & |\mathbf{B}|\cos 45^\circ & 0 \end{vmatrix} \\
&= \hat{\mathbf{x}}[(0)(0) - (-3.50)(|\mathbf{B}|\cos 45^\circ)] - \hat{\mathbf{y}}[(0)(0) - (-3.50)(-|\mathbf{B}|\sin 45^\circ)] \\
&\qquad + \hat{\mathbf{z}}[(0)(|\mathbf{B}|\cos 45^\circ) - (0)(-|\mathbf{B}|\sin 45^\circ)] \\
&= 3.50|\mathbf{B}|\cos 45^\circ\,\hat{\mathbf{x}} + 3.50|\mathbf{B}|\sin 45^\circ\,\hat{\mathbf{y}} \\
&= 3.50|\mathbf{B}|(\cos 45^\circ\,\hat{\mathbf{x}} + \sin 45^\circ\,\hat{\mathbf{y}}) \\
&= 3.50|\mathbf{B}|\left(\frac{\hat{\mathbf{x}}}{\sqrt{2}} + \frac{\hat{\mathbf{y}}}{\sqrt{2}}\right) \\
&= \frac{3.50|\mathbf{B}|}{\sqrt{2}}(\hat{\mathbf{x}} + \hat{\mathbf{y}}) \\
&= \frac{3.50|\mathbf{B}|}{\sqrt{2}}\langle 1, 1, 0\rangle \text{ cm}^2
\end{aligned}$$

Consequently, $\mathbf{A} \times \mathbf{B}$ also lies in the xy-plane but perpendicular to $\mathbf{B}$.